RADIANT-CONDUCTIVE HEAT TRANSFER IN A PLANE AMMONIA LAYER

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Results of experimental and theoretical investigations of radiant-conductive heat transfer in a plane ammonia layer are presented.

A number of theoretical and applied studies related with calculations of complex heat transfer [1-5] and the solution of certain auxiliary problems pertinent to thermophysical investigations are devoted to an investigation of heat transfer in plane layers of absorbent media. At the same time there are few experimental studies along this line.

Here we will present an experimental and theoretical examination of radiant-conductive heat transfer in an approximation of one-dimensional heat transfer in a plane layer of absorbent gas.

The physical scheme of the experiment amounts to the following. Two metal plates made in the form of thin disks are arranged in plane-parallel planes and form the layer of investigated medium. The plates are maintained in an isothermic state, the upper plate being heated and the lower one being cooled (Fig. 1). The isothermicity of the plates and layer in the planes of the parallel plates is checked by means of thermocouples embedded in the plates and by special nichrome–constantan thermocouples (diameter 0.06 mm) moving within the layer (Fig. 1). This gave grounds to assume that the steady flow of heat throughout the layer is one-dimensional. The selection of the distance between plates which excludes free convection in the space between them was determined indirectly from the results of measuring the temperature distribution in the layer of a diathermic medium (linear character of distribution) under various pressure conditions (from $P \sim 40$ mm Hg to $P \sim 2$ abs. atm).

The position of the movable thermocouples relative to the bounding surfaces was determined by a KM6 cathetometer. The temperature level of the heated plate was maintained by regulating the voltage supplied to the heater. The chamber was preliminarily evacuated ($\sim 5 \cdot 10^{-2}$ mm Hg) before filling with the investigated gas.

The experimental investigation amounted to measuring the temperature distribution of the gas (from the readings of the near-axial thermocouple) as a function of the optical thickness of the medium and of the temperature difference between plates.

To conduct standard measurements and to estimate the errors due to irradiation of the thermocouples, a diathermic medium (air) was enlisted in the first study. In this case the process of heat transfer in the layer is determined by molecular heat conductivity. The solution of the equation of heat conductivity

$$\frac{d}{dy}\left(\lambda \frac{dT}{dy}\right) = 0, \qquad (1)$$
$$T(0) = T_1, \quad T(\delta) = T_2$$

in the case of validity of the linear character of the temperature dependence of the coefficient of heat conductivity

$$\lambda = a + bT \qquad (2)$$

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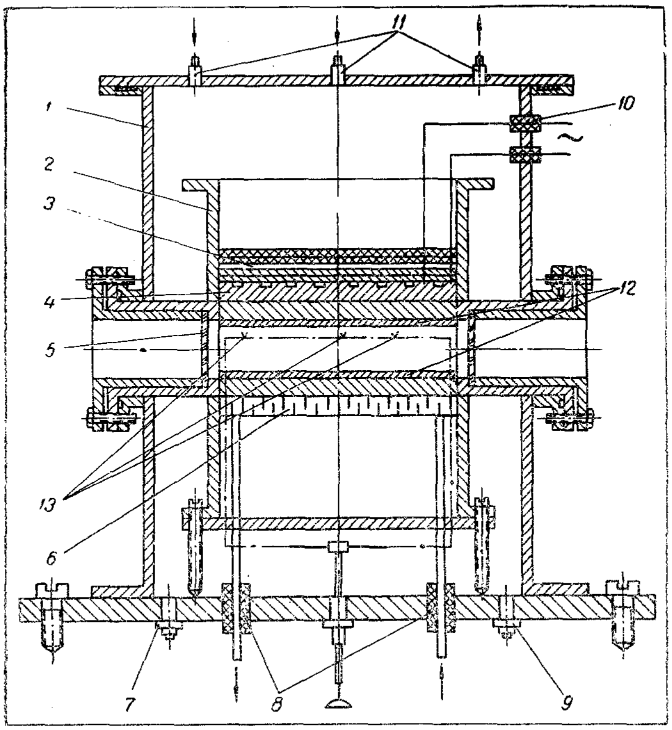

Fig. 1. Diagram of experimental device: 1) exterior cylindrical casing; 2) interior casing; 3) insulation of layers of glass fabric and stainless steel foil; 4) heater; 5) Plexiglas observation ports; 6) refrigerator (water); 7) lead to backing pump; 8) water inlet and outlet with rubber seals; 9) lead for thermocouples; 10) conductor to heater; 11) outlets for gas; 12) experimental plates forming the investigated gas layer; 13) movable thermocouples.

is written

$$T = -\frac{a}{b} \pm \sqrt{\left(\frac{a}{b}\right)^2 + A}, \tag{3}$$

$$A = \frac{2a}{b}\left[T_1 - \zeta\,(T_1 - T_2)\right] + T_1^2 - \zeta\,(T_1^2 - T_2^2).$$

Figure 2 presents the results of calculating the temperature distribution ($\vartheta = (T - T_2)/(T_1 - T)$) in the layer of air at $T_2 = 508$°K.

The experimental results, indicating good agreement with the calculation, are also presented there. The slight divergence due to irradiation of the bead of the thermocouples owing to its small size and low temperature level proved to be insignificant.

In the case of the absorbent medium the indicated error is apparently smaller owing to partial shielding of radiation of the heated surface by the medium.

As an absorbent medium we used gaseous ammonia NH_3 having good absorptivity. The absorption coefficient $\varkappa$ is determined by the total emittance of the layer, and the processes of heat transfer by radiation are considered in an approximation of a model of a gray medium.

Radiant-conductive heat transfer in a plane layer of a gray heat-conducting body is described by a nonlinear integral equation representing the formal solution of the nonlinear integro-differential equation of energy

$$-\frac{d}{dy}\left(\lambda \frac{dT}{dy}\right) + \frac{dE}{dy} = 0, \tag{4}$$

$$T(0) = T_1, \; T(\delta) = T_2.$$

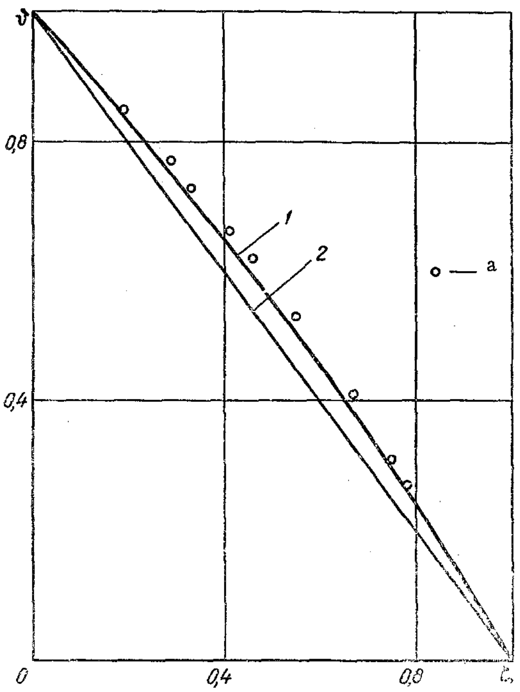

Fig. 2

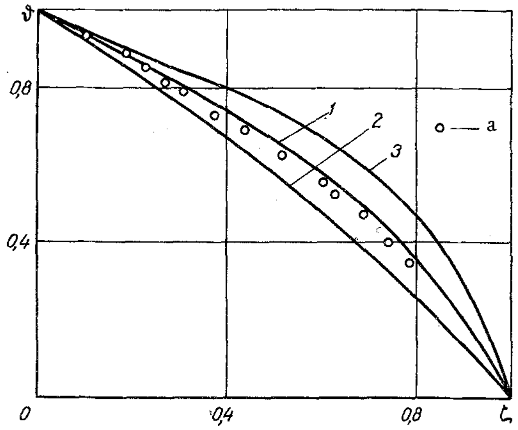

Fig. 3

Fig. 2. Distribution of dimensionless temperature ϑ in a layer of a diathermic medium (air): a) experiment (t = 235°C); 1) solution by (3) (a/b = 56.6); 2) solution at λ = const.

Fig. 3. Distribution of dimensionless temperature ϑ in a layer of an absorbent NH_3 medium (P = 1 abs. atm, δ = 12 mm): a) experiment ($\theta_1 = 0.6$; $\theta_2 = 1.0$; $N_{\lambda_* \varkappa} = 0.026$; $a/\lambda_* = -0.347$; $bT_*/2a = -1.93$; $h_0 = 0.15$); 1) solution of (5) under the experimental conditions; 2) the same, for λ = const ($\theta_1 = 0.6$; $\theta_2 = 1.0$; b = 0; $\lambda = a$ at t = 0°C; $h_0 = 0.15$); 3) solution of (5) for $h_0 = 0.30$ under the experimental conditions.

Here E is the hemispheric density of the resultant radiation flux, determined in conformity with the radiating system being considered by the integral relation presented in [8].

On the assumption of a linear character of the temperature dependence of the coefficient of molecular heat conductivity, the temperature distribution in the layer is determined by solving the nonlinear integral relation:

$$\theta(h) = a(h) + \frac{1}{2N}\int_0^{h_0} \theta^4(\xi)\, G(h, \xi)\, d\xi - \frac{bT_*}{2a}\,\theta^2(h), \tag{5}$$

$$a(h) = \theta_1 - \frac{\theta_1 - \theta_2}{h_0} h + \frac{bT_*}{2a}\left(\theta_1^2 - \frac{\theta_1^2 - \theta_2^2}{h_0} h\right) + \frac{1}{2N}\left(\frac{A_1}{D}\,\theta_1^4 \chi_1(h) + \frac{A_2}{D}\,\theta_2^4 \chi_2(h)\right), \tag{6}$$

obtained from (4) by successive integration with consideration of the boundary conditions. The geometric–optical parameters of radiation $\chi_1(h)$, $\chi_2(h)$, $G(h, \xi)$, and D, representing functionals of the optical properties of the boundaries A, R of the medium h, h_0 and of the exponential integrals K_n, are determined by:

$$G(h, \xi) = K_3(\xi) - K_3|h - \xi| + \frac{h}{h_0}(K_3(h_0 - \xi) - K_3\xi)) + 2\frac{R_1}{D}\chi_1(h)\,K_2(\xi) + 2\frac{R_2}{D}\chi_2(h)\,K_2(h_0 - \xi), \tag{7}$$

$$\chi_1(h) = \frac{1}{3} - K_4(h) - \frac{h}{h_0}\left(\frac{1}{3} - K_4(h_0)\right) + 2R_2 K_3(h_0)\left[K_4(h_0) - K_4(h_0 - h) + \frac{h}{h_0}\left(\frac{1}{3} - K_4(h_0)\right)\right], \tag{8}$$

$$\chi_2(h) = K_4(h_0) - K_4(h_0 - h) + \frac{h}{h_0}\left(\frac{1}{3} - K_4(h_0)\right) + 2R_1 K_3(h_0)\left[\frac{1}{3} - K_4(h) - \frac{h}{h_0}\left(\frac{1}{3} - K_4(h_0)\right)\right], \tag{9}$$

$$D = 1 - 4R_1 R_2 K_3^2(h_0). \tag{10}$$

The total radiant-conductive flux is determined by the following integral relation:

$$\Phi = b(h_0) + \frac{2}{h_0}\int_0^{h_0} \theta^4(h)\,G_1(h_0, h)\,dh\,, \tag{11}$$

$$b(h_0) = \frac{4}{h_0}\left[N(\theta_1 - \theta_2) + \frac{bT_*}{2a}N(\theta_1^2 - \theta_2^2) + \frac{1}{2}\left(A_1(h_0)\,\theta_1^4 - A_2(h_0)\,\theta_2^4\right)\right], \tag{12}$$

$$G_1(h_0, h) = K_3(h) - K_3(h_0 - h) + 2A_1(h_0)\frac{R_1}{A_1}K_2(h) - 2A_2(h_0)\frac{R_2}{A_2}K_2(h_0 - h), \tag{13}$$

$$A_1(h_0) = \frac{A_1}{D}\left(\frac{1}{3} - K_4(h_0)\right)(1 - 2R_2K_3(h_0)),\ A_2(h_0) = \frac{A_2}{D}\left(\frac{1}{3} - K_4(h_0)\right)(1 - 2R_1K_3(h_0))\,. \tag{14}$$

Integral equation (5) is solved numerically by Newton's iterative method. In this case the linear distribution of temperature $\theta = \theta_1 + [h(\theta_1 - \theta_2)/h_0]$ was used as the initial approximation.

Figure 3 presents the results of comparing the numerical solution of Eq. (5), transformed to the form $\vartheta = \vartheta(\xi)$, with the experiment in conformity with the measurements of the temperature distribution in the NH_3 layer at P = 1 abs. atm of thickness $\delta = 12$ mm formed by two plates, of which the upper, heated plate radiates well (stainless steel, $A_2 = 0.85$) and the lower, cooled plate reflects effectively (brass, $A_1 = 0.15$).

The absorption spectrum of NH_3 has two characteristic regions in the 6 and 10 μ range [9]. According to Wein's displacement law this corresponds to a radiation peak of the upper plate at temperatures of ~500°K and ~300°K, respectively. In this connection the temperature of the upper plate in the experiment was maintained at the level $T_2 = 500$°K ($T_1 = 290$°K). The total emittance of the layer ($\varepsilon = 1 - \exp(-\varkappa\delta)$), determined in [10] as a function of T and $p\delta$, is equal to ~0.15. The optical thickness of the layer $h_0 = \varkappa\delta = 0.15$. In the temperature being considered, according to [11],

$$\frac{a}{\lambda_*} = -0.347,\quad \frac{bT_*}{2a} = -1.93,\quad N_{\lambda_*\varkappa} = \frac{\lambda_*\varkappa}{4\sigma_0 T_*^3} = 0.026.$$

In this case the temperature of the heated surface $T_* = T_2$ is taken as the characteristic temperature. The satisfactory agreement between the experimental measurement and the results of the numerical solution confirm the validity of the heat-transfer model described by integral equation (5).

Figure 3 also presents the results of solving (5) for $h_0 = 0.15$, $\theta_1 = 0.6$, $\theta_2 = 1.0$, and $\lambda = a$ (t = 0°C, b = 0). They attest to the fundamental importance of taking the function $\lambda = \lambda(T)$ into account.

The results of the solution of Eq. (5) for parameters determined by the experimental conditions but for $h_0 = 0.30$ reflect the essential role of the optical density of the layer of the medium in the temperature distribution.

The mean-free path of a photon $1/\varkappa$ as applied to the experimental conditions exceeds the dimensions of the layer, but the process of multiple reflections from the surface of the lower plate slightly increases the effective dimensions of the layer and therefore increases the probability of collisions within the absorbent medium.

In connection with this, the effective heat conductivity as applied to cases of low optical density should be analyzed with consideration of the real values of the optical properties of the boundaries of the investigated layer of the medium.

We can show that in the case of small temperature drops the expression for the effective heat conductivity can be represented in a closed form.

The expression for the density of a radiant-conductive heat flux is written as:

$$q = -\lambda\varkappa\frac{dT}{dh} - E(h) \tag{15}$$

or (after integration and consideration of the boundary conditions) as

$$q = -\lambda\varkappa\frac{T_1 - T_2}{h_0} - \int_0^{h_0} E(h)\,dh. \tag{16}$$

Using the expression for the hemispherical density of the resultant radiation in [8] and integrating it by parts, we represent (16) in the following form:

$$q = -\lambda\varkappa \frac{T_1 - T_2}{h_0} - \frac{2}{h_0}\int_0^{h_0}\int_0^{h_0} G(h, \xi)\frac{dE_0(\xi)}{d\xi}\, d\xi\, dh. \tag{17}$$

Setting $\frac{dE_0(\xi)}{d\xi} = \frac{dE_0(h)}{dh}$ = const (linear character of the radiation distribution in the layer of absorbent medium) and $\lambda\varkappa \frac{T_1 - T_2}{\varkappa\delta} \sim \lambda \frac{dT}{dy}$, we obtain:

$$q = -\lambda\frac{dT}{dy} - \frac{2}{h_0}\frac{dE_0}{dT}\frac{dT}{dh}\int_0^{h_0}\int_0^{h_0} G(h, \xi)\, d\xi\, dh, \tag{18}$$

or

$$q = -\lambda\frac{dT}{dy} - \frac{16\sigma_0}{3\varkappa}T^3 F(\varkappa\delta, R_1, R_2)\frac{dT}{dy}. \tag{19}$$

Here

$$F(\varkappa\delta, R_1, R_2) = \frac{2}{3}h_0\int_0^{h_0}\int_0^{h_0} G(h, \xi)\, d\xi\, dh$$

$$= 1 - \frac{3}{h_0}\left[\frac{1}{4} - K_5(h_0) - \left(\frac{1}{3} - K_4(h_0)\right)^2\left(\widetilde{A}_1(h_0)R_1 + \widetilde{A}_2(h_0)R_2\right)\right]; \tag{20}$$

$$\widetilde{A}_1(h_0) = \frac{1}{D}\left(1 - 2R_2K_3(h_0)\right),\ \widetilde{A}_2(h_0) = \frac{1}{D}\left(1 - 2R_1K_3(h_0)\right). \tag{21}$$

Thus the expression for the effective heat conductivity

$$\lambda_{ef} = \lambda + \frac{16\sigma_0}{3\varkappa}T^3 F(\varkappa\delta, R_1, R_2) \tag{22}$$

reflects, besides the optical thickness of the medium, the effect of the optical properties of the radiating boundaries.

When $R_1 = R_2 = 0$ the results of calculations by (20) agree with similar calculations made in [6] on the assumption of a sufficiently large optical thickness of the layer.

An analysis of the experimental results on determination of the coefficients of heat conductivity of dropping liquids [6] indicates a substantial effect of reemission in the investigated media. In this connection, the determination of the coefficient of effective heat conductivity, which within a small temperature interval in an absorbent and refracting medium is written as

$$\lambda_{ef} = a + bT + \frac{16}{3}\cdot\frac{n^2}{\varkappa}\sigma_0 TF^3(\varkappa\delta, R_1, R_2), \tag{23}$$

acquires special importance. The results of calculating the radiant-conductive heat flux ($q = -\lambda_{ef}(\Delta T/\delta)$) with consideration of (23) with respect to the average temperature value of the medium ($T = \overline{T}$) agree well with the numerical calculations by (11).

NOTATION

$\vartheta = (T - T_2)/(T_1 - T_2)$,
$\theta = T_i/T_*$ are the dimensionless temperatures;
$N = N_{\lambda_*\varkappa}(a/\lambda)$;
$N_{\lambda_*\varkappa} = \lambda_*\chi/4\sigma_0 T_*^3$ is a dimensionless radiation–conduction parameter;

$K_n(x) = \int_0^1 \exp\left(-\frac{x}{\mu}\right) \mu^{n-1} \frac{d\mu}{\mu}$ is an exponential integral;

A_i, R_i are the absorptance and reflectance of the gray bounding surfaces;
$\Phi = q/\sigma_0 T_*^4$ is the dimensionless radiant-conductive flux;
$E_0 = \sigma_0 T^4$ is the density of equilibrium radiation;
δ is the thickness of the layer of the medium;
χ is the absorption coefficient;
$h_0 = \chi\delta$, $h = \varkappa y$ are the optical thickness and depth of the layer, respectively;
$\zeta = y/\delta = h/h_0$.

Subscripts

$i = 1, 2$ is the numeration of the boundaries;
$*$ is the subscript of the characteristic parameter.

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